Decoupling in Distillation

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Decoupling of multivariable control systems can have advantages. Setpoint changes and servo-control are generally easier performed for a decoupled process than for an interacting one. The calculation and realization of feedforward controllers are usually simplified if the system is decoupled.

Due to the dynamic characteristics of the process, two-composition control of continuous distillation is well suited for decoupling. Schemes obtained by very simple calculations utilizing partial decoupling may compare quite favorably with schemes obtained by optimal control theory (see Waller et al., 1974).

A paper by Luyben (1970) on distillation decoupling has been paid considerable attention. Niederlinski (1971) showed that advantages can be taken of interaction in order to attenuate disturbances. Changlai and Ward (1972) discussed Luyben's schemes on the basis of previous work. Toijala and Fagervik (1972) showed the generality of some of Luyben's results, and Luyben and Vinante (1972) followed up the theory experimentally.

DECOUPLING THEORY

To decouple a process with the transfer function matrix **G**, a decoupling matrix **D** is chosen so that

$$\mathbf{GD} = \mathbf{T} \tag{1}$$

is diagonal (Figure 1). The scheme is illustrated in Figure 2 for a system with two inputs and two outputs controlled by two primary feedback controllers **P**, where

$$\mathbf{D} = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \tag{2}$$

The matrix **D** is obtained from (1) as $\mathbf{D} = \mathbf{G}^{-1}\mathbf{T}$, provided that $\mathbf{G}^{-1}$ exists. With the matrices given in (2) and

$$\mathbf{T} = \begin{bmatrix} T_{11} & 0 \\ 0 & T_{22} \end{bmatrix}$$

D is given by

$$\mathbf{D} = \frac{1}{G_{11}G_{22} - G_{12}G_{21}} \begin{bmatrix} G_{22}T_{11}, & -G_{12}T_{22} \\ -G_{21}T_{11}, & G_{11}T_{22} \end{bmatrix} \tag{3}$$

There are two general approaches to solve this control problem. One is to choose the **T** matrix and then realize, if possible, the elements of the resulting **D** matrix (Changlai and Ward, 1972). A common choice in this approach (Zalkind, 1967) is $T_{11} = T_{22} = 1$. It is hard to see the merits of this choice. Zalkind (1967) comments on it with the words "the non-interacting controller network (**D**) usually carries too much load, leaving the conventional controllers (**P**) with nothing to do."

A more natural choice is $T_{11} = G_{11}$ and $T_{22} = G_{22}$. This choice is labeled *ideal decoupling* by Luyben (1970). As seen from Equation (3) this choice leads to rather complicated expressions for the elements of **D**, which then often are difficult to realize. A study of the general charac-

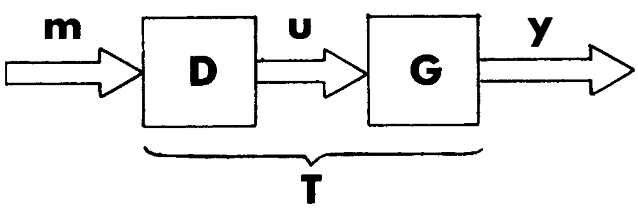

Fig. 1. Decoupling of process G.

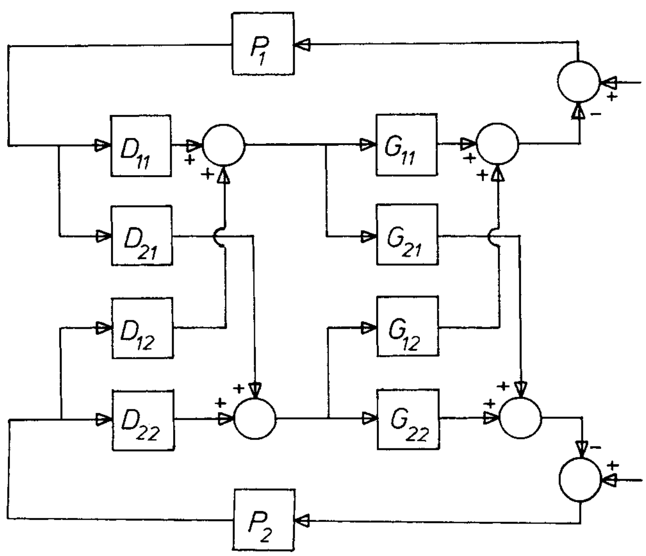

Fig. 2. Decoupling of process with two inputs and two outputs.

teristics of the distillation process (Toijala and Jonasson, 1970; Toijala, 1971) reveals that this is the case in distillation, a result supported by Luyben's Figure 2 (1970).

The other general approach is to choose some of the elements of **D** and then tune the feedback controllers **P** according to the resulting **T**. This choice seems to be the more logical one. The choice can usually be made so as to give easily realizeable decoupling elements. Since D_{12} and D_{21} are principally feedforward controllers, this approach more closely follows the common principle that the feedforward controllers do the rough corrections of the disturbances, whereas the feedback controllers correct the errors caused by the inaccuracy of the feedforward control.

As shown by Equation (3), two elements of **D** can be arbitrarily chosen. (Generally, n elements of a system with n inputs and n outputs may be freely chosen.) A common choice is $D_{11} = D_{22} = 1$ (Zalkind, 1967; Niederlinski, 1971), labeled *simplified decoupling* by Luyben (1970). With this choice the **D** matrix becomes

$$\mathbf{D} = \begin{bmatrix} 1 & -\dfrac{G_{12}}{G_{11}} \\ -\dfrac{G_{21}}{G_{22}} & 1 \end{bmatrix} \tag{4}$$

In general the off-diagonal decoupling elements $-G_{21}/G_{22}$ and $-G_{21}/G_{11}$ [not $-G_{21}/G_{11}$ and $-G_{12}/G_{22}$ as stated by Zalkind (1967)] are easier to realize than the decoupling elements obtained by choosing simple elements in **T**. For continuous distillation $-G_{21}/G_{22}$ and especially $-G_{12}/G_{11}$ are often frequency independent up to quite high frequencies (Toijala and Fagervik, 1972). This is also illustrated by Luyben's Figure 6 (1970).

The choice of two elements in **D** equal to unity means that there are only two decoupling elements to be realized. This is a considerable advantage. A drawback is that the off-diagonal elements of (4) may be physically impossible to realize. This drawback, however, might be eliminated through another choice of **D**, which still retains the advantage of (4) that two decoupling elements are equal to unity. This fact seems to have been somewhat overlooked in the literature. Actually, any two elements of **D** in (3) may be chosen equal to unity, as long as they are not in the same column in the **D** matrix. This is also obvious after a glance at Figure 2. Thus, in addition to (4) there are the following three possibilities for a system with two inputs and two outputs

$$\mathbf{D} = \begin{bmatrix} 1, & 1 \\ -\dfrac{G_{21}}{G_{22}}, & -\dfrac{G_{11}}{G_{12}} \end{bmatrix} \tag{5}$$

$$\mathbf{D} = \begin{bmatrix} -\dfrac{G_{22}}{G_{21}}, & -\dfrac{G_{12}}{G_{11}} \\ 1, & 1 \end{bmatrix} \tag{6}$$

$$\mathbf{D} = \begin{bmatrix} -\dfrac{G_{22}}{G_{21}}, & 1 \\ 1, & -\dfrac{G_{11}}{G_{12}} \end{bmatrix} \tag{7}$$

This has interesting and important consequences. If a nonunity element of (4), say $-G_{21}/G_{22}$, is difficult to realize, it is usually easy to realize its inverse, $-G_{22}/G_{21}$, and use (6) [or (7)] instead of (4), etc.

It should be noted that the closed loop responses are different for (4), (5), (6), and (7), a fact that should be taken into account when tuning the controllers **P**.

APPLICATION TO DISTILLATION

In two-composition control of distillation columns a sensor (usually) in the enriching section is used in a feedback control scheme with the reflux flow or distillate rate as the manipulative variable. A second sensor (usually) in the stripping section is used for manipulation of heat input to the reboiler. If the reflux and vapor flow rates are used for control, the transfer functions of **G** (neglecting dynamics of piping, valves, and reboiler) are, for example,

$$G_{11} = x_e/\Delta L, \quad G_{21} = x_s/\Delta L, \quad G_{12} = x_e/\Delta V, \quad G_{22} = x_s/\Delta V \tag{8}$$

In binary distillation (Toijala and Jonasson, 1970; Toijala, 1971) and to a large extent in multi-component distillation also (Toijala and Gustafsson, 1972), the transfer functions of (8) are, with flow lags neglected, close to first order. Furthermore, the time constants of G_{12} and G_{11} are often approximately equal, the same being true as regards G_{21} and G_{22}, too. The realization of the elements of (4) to (7) (through a lead-lag network or simply a gain) is thus easy up to quite high frequencies where secondary effects like flow lags become important.

Usually the liquid flow lags are considerably larger than the vapor flow lags. Since the number and magnitude of flow lags are larger in G_{21} than they are in G_{22}, the decoupling element $-G_{21}/G_{22}$ is easier to realize than its inverse. Although the dynamics of valves, reboiler, and piping is to be included in the transfer functions of **G** (see Toijala and Fagervik, 1972), these lags do not usually make $-G_{22}/G_{21}$ easier than its inverse to realize. Consequently, it is a fairly general rule in distillation that the schemes (4) or (5) should be used, not (6) or (7).

No general rule can be given for the choice between (4) and (5), that is, whether $-G_{12}/G_{11}$ or its inverse is easier to realize. The liquid flow lags are larger than the vapor flow lags, but the number of vapor flow lags is usually considerably larger than the number of liquid flow lags in the element $-G_{12}/G_{11}$. However, the column flow lags in this element are sometimes so small that lags in piping, reboiler, etc., become determining.

Two numerical examples chosen from the literature will illustrate the general discussion above. In a simulation study Luyben (1970) obtained $-G_{21}/G_{22} = 0.95\, e^{-1.5 \text{ min } s}/(0.4 \text{ min } s + 1)$ and $-G_{12}/G_{11} = 0.95$. Thus, in this case (4)and (5) can be realized with the same ease.

In an experimental study by Luyben and Vinante (1972) on a 24-plate bubble-cup column the sensors were located at plates 4 and 17 (counting from below). The transfer functions of (8) (with secondary effects included) were found to be approximately

$$G_{11} = -2.2\, e^{-1 \text{ min } s}/(7 \text{ min } s + 1),$$

$$G_{12} = 1.3\, e^{-0.3 \text{ min } s}/(7 \text{ min } s + 1),$$

$$G_{21} = -2.8\, e^{-1.8 \text{ min } s}/(9.5 \text{ min } s + 1), \text{ and}$$

$$G_{22} = 4.3\, e^{-0.35 \text{ min } s}/(9.2 \text{ min } s + 1).$$

Since the dead times in G_{12} and G_{22} are nearly equal, the vapor flow lags seem to have been too small to be detected. The dead time measured for these transfer functions is probably mainly caused by the reboiler system. The dead-time difference between G_{11} and G_{21} is 0.8 min. and the number of plates between the two sensors is 13. These figures would give liquid flow lags of about 4 sec/plate, a reasonable value (Toijala, 1971). Since there are only 7 liquid flow lags included in the 1 min. dead time of G_{11}, there is an additional dead time of about 0.5 min. in the reflux flow system, probably a transportation lag mainly caused by the piping between the column and the reflux drum. With the transfer functions given above, the decoupling elements of (4) become approximately:

$$-G_{21}/G_{22} = 0.6\, e^{-1.5 \text{ min } s}/(0.2 \text{ min } s + 1)$$

and

$$-G_{12}/G_{11} = 0.6\, e^{+0.7 \text{ min } s}\,.$$

It is evident that $-G_{21}/G_{22}$ is physically realizeable whereas $-G_{12}/G_{11}$, having a negative dead time, is not. However, the inverse $-G_{11}/G_{12} = 1.7\, e^{-0.7 \text{ min } s}$, is easily realizeable. Thus, in this case (5) is the one of the schemes (4) to (7) that should be used.

DISCUSSION

Choosing $T_{11} = G_{11}$ and $T_{22} = G_{22}$, Equation (3) becomes

$$\mathbf{D} = \frac{G_{11}G_{22}}{G_{11}G_{22} - G_{12}G_{21}} \begin{bmatrix} 1\,, & -\dfrac{G_{12}}{G_{11}} \\ -\dfrac{G_{21}}{G_{22}}, & 1 \end{bmatrix}$$

Equation (9) is used by Luyben (1970) in his ideal decoupling scheme, whereas Equation (4) is used in his simplified decoupling scheme. A comparison of Equations (4) and (9) shows that if the ideal decoupling elements are known, the simplified elements are directly obtained. Thus Luyben's Figure 6 is directly obtained from his Figure 2 and the corresponding analytical expressions for the simplified elements from the corresponding ideal elements.

The element $D_{11} = D_{22}$ in ideal decoupling shown in (9) to be $G_{11}G_{22}/(G_{11}G_{22} - G_{12}G_{21})$, has a gain which in Luyben's example strongly increases with product purity. This causes instability at high product purity for the ideal decoupling scheme whereas the simplified decoupling scheme is stable. From this fact Luyben draws the conclusion that ideal decoupling may be of limited applicability. The motive for this conclusion is somewhat misleading. Ideal decoupling may be less preferable because the decoupling elements are complicated and difficult to realize especially at high frequencies, whereas the elements of simplified decoupling are generally not. But it should be emphasized that instability results because the settings for the feedback controllers are the same in both schemes in Luyben's example. Looking at the ideal decoupling scheme as it is drawn in Figure 3, it is seen that the **P**-controllers in the ideal decoupling scheme have to handle the elements D_{11} and D_{22} in addition to the process and decoupling elements of the simplified decoupling scheme. In Luyben's Case 2/98, which is unstable with the ideal scheme, $D_{11} = D_{22} = 10/(12.5 \text{ min } s + 1)$. Quite naturally, the controller settings used for simplified decoupling should be altered when these elements are included in the loops. Moreover, the phase lags caused by D_{11} and D_{22} put an extra burden on the **P**-controllers, why the scheme ideal decoupling in this case is less attractive than simplified decoupling, even if the controller settings are corrected so as to give a stable response.

It is interesting and challenging that relatively small changes of, for example, the gains of the decoupling elements may increase or considerably decrease control quality (Toijala and Fagervik, 1972). Partial decoupling may thus be better than complete decoupling, but there is also quite a risk of drastically decreasing the controllability properties of the process through the introduction of the decoupling elements.

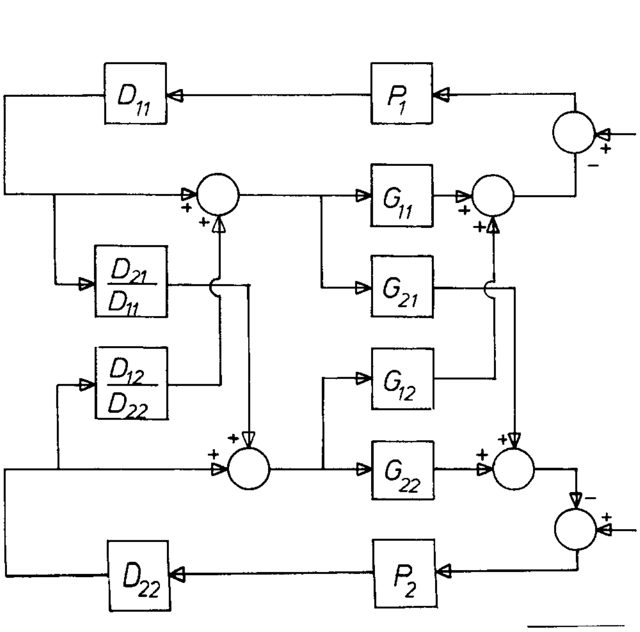

Fig. 3. Alternative drawing of Figure 2.

NOTATION

- $\mathbf{D}$ = decoupling matrix
- D = decoupling element, Equation (2), Figure 2
- $\mathbf{G}$ = process transfer function matrix
- G = process transfer function, Equation (2), Figure 2
- ΔL = change in reflux flow rate
- $\mathbf{P}$ = primary feedback controllers
- P = primary feedback controller
- s = complex variable
- $\mathbf{T}$ = matrix, Equation (1)
- T = element of T
- ΔV = change in vapor flow rate
- x = liquid composition change
- e = enriching section
- s = stripping section

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